THE ANALYSIS AND DESIGN OF SHOCK ABSORBERS WITH CONSTANT DECELERATION FOR VARIABLE LOADING

BY

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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1980

To my wife Helen. Without her love, confidence, and patient encouragement, my graduate studies would never have been completed

ACKNOWLEDGMENTS

[illegible] for guiding my [illegible] as [illegible] Chairman

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NOMENCLATURE

[illegible]

[illegible] acceleration - ft./sec./sec.

[illegible] head feet

[illegible] constant - lbs./ft

m mass slugs

n a parameter such that $W = nW_1 + W + 1$

o zero

p hydraulic pressure lbs./sq. ft.

r regulator subscript

t time - seconds

v velocity - ft./sec., valve subscript

w specific weight - lbs./cu. ft

x displacement - ft

ρ specific mass - slugs/cu. ft

A pistons area - sq. ft

C_d orifice discharge coefficient

D damping force - lbs

F external force - lbs

F cushion force - lbs

I vehicle loading - lbs.

N deceleration load factor

[illegible]

[illegible]

t closure time - sec.

V_0 impact velocity - ft /sec

W weight of empty vehicle - lbs

X cushion length - ft

Abstract of Dissertation Presented to the Graduate Council of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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December, 1980

Chairman: Dr. Knox T. Millsaps

Major Department: Engineering Sciences

For a given impact velocity of a vehicle striking a fixed surface [illegible]ing damped shock absorber is developed to provide the same maximum [illegible]tion of the vehicle irrespective of the loading at impact. The [illegible] damping and external forces are generalized (linear or nonli[illegible] [illegible] the initial part of the analysis. For a shock absorber with [illegible]ciency, the maximum vehicle deceleration is shown to be a [illegible] only the impact velocity and the cushion length. The sum [illegible] damping and external forces is found to be a constant [illegible] function of the vehicle weight.

[illegible]llustrate the method of design analysis, a shock absorber

[illegible] absorber [illegible] and [illegible] parallel spring is analyzed for a number of design case examples of horizontal and vertical vehicle motion at impact. Orifice area equations are developed, and dimensionless plots versus cushion length and closure time are presented. An automatic orifice area regulator is then designed to provide optimum shock absorber response for all loading conditions of a freight car at impact. The time response of the regulated orifice area is a maximum for the car with maximum load. At impact it is shown to be approximately 6% of the shock absorber closure time.

INTRODUCTION

In some industrial applications, a shock absorber is used to dissipate and/or absorb the energy of a moving mass or vehicle with a variable load [1 - 14]. At present, most shock absorbers are designed to provide a constant cushion force over the full cushion length of the shock absorber for only the maximum weight and impact velocity of the vehicle. For a partially loaded vehicle, the cushion force will have the same maximum value, but the force is no longer a constant. When the entire cushion length of the shock absorber is not utilized, the maximum deceleration of the vehicle increases, and this increase is usually undesirable.

If the full cushion length could be utilized for all vehicle loading conditions, a more efficient shock absorber could be designed to provide the optimum response of the same constant value of maximum deceleration for all variations of the vehicle load [13, 14].

The purpose of the present dissertation is to show how such a self-regulating shock absorber can be designed to provide this conditionally better response.

ANALYSIS

Performance Specifications

Most products are designed to satisfy a number [illegible] to initial cost, performance, reliability, maintenance [illegible] requirements or specifications are usually defined and [illegible] of any serious design effort

The performance specifications for the subsequent [illegible] design of a vehicle shock absorber were selected to [illegible] absorber would have a constant deceleration for all loading [illegible] of the vehicle at the specified maximum impact velocity. The maximum cushion length of the shock absorber is also specified

The above specifications seem quite simple, however, it may not be possible to satisfy all of them. The analysis will determine if the simultaneous satisfaction of the specification is possible.

Genera[illegible] Analy[illegible]

The analysis is started by assuming [illegible] will provide the desired cushio[illegible] [illegible]

[illegible] weight of empty vehicle

[illegible] = maximum weight o[illegible] load

[illegible] parameter [illegible]

[illegible]

[illegible]

[illegible] with an input [illegible] applied to the vehicle.

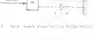


Figure 1 Mass shock absorbe[illegible]

[illegible] spring force [illegible]

[illegible]

external forcing function. The cushion force F(e,x) acting on the mass mm of this generalized system will be

$$F(e,x) = E(e,x) - D(e,x) - S(e,x) \tag{1}$$

let

$$a(n,x) = \text{acceleration of mm at the displacement } x$$

The cushion force will then be

$$F(e,x) = mma(n,x) \tag{2}$$

The kinetic energy at impact plus the work done by the forcing function must be equal to the work absorbed or dissipated over the cushion length X of the shock absorber. Therefore

$$\tfrac{1}{2}mmV^2 + \int_0^X E(n,x)dx = \int_0^X D(n,x)dx + \int_0^X S(n,x)dx \tag{3}$$

or

$$\tfrac{1}{2}mmV^2 + \int_0^X F(e,x)dx = 0 \tag{4}$$

It is desired to minimize the maximum cushion force for each value of the mass mm at impact. Since mm is a constant over the cushion length the cushion force is considered to be optimized when it becomes a

[illegible] be [illegible] closure [illegible] of the [illegible]

$$[illegible] \qquad (5)$$

[illegible] integrated [illegible] displacement [illegible] the integration is [illegible] cushion [illegible]

$$[illegible] \qquad (6)$$

[illegible]) may [illegible] the equation for [illegible] as a [illegible] cushion length independent of [illegible] and the vehi[illegible] loading. The optimum deceleration is

$$[illegible] \qquad (7)$$

[illegible] of the optimum displacement x, time t and velocity [illegible] obtained.

$$[illegible] \qquad (8)$$

$$[illegible] \qquad (9)$$

$\dot{x}(t) = V(1 - $ [illegible] (10)

$\dot{x}(t) = V(1 - x$ [illegible] (11)

The optimum response or output [illegible] therefore be defined by any one of equations (6) through [illegible]

To obtain this output, the cushion force must vary with the mass as shown by equation (6). Therefore, the type of damping and parallel spring must be selected with the restriction that the sum of their forces can be varied in such a manner as to optimize the cushion force for any and all vehicle loading conditions at impact. In addition, a shock absorber control will be required to adjust the cushion force, either manually or by means of a self-contained regulator.

The manual or open loop control (15, 16) requires adjustment prior to impact, for each vehicle loading condition. All open loop controls also have the disadvantage that the accuracy with which the input is adjusted determines the output. Shock absorbers with this type control are now available (13, 14), but because of the above disadvantages they have very limited application and will not be discussed further.

For the regulator or closed loop type of feedback control, the output of the shock absorber can be used to control and optimize the cushion force for all vehicle loading conditions at impact. The block diagram of such a control is shown in Figure 2.

[illegible]

[illegible] is selected [illegible] compatible with [illegible] may be eliminated as show [illegible]

[illegible] 3 Simplified Feedback Control

[illegible] preliminary and final design analyses that follow, the [illegible] of the feedback control will be identified

[illegible]

[illegible] Sprin[illegible]

[illegible] stripped

[illegible] to be the quadratic law dampin[illegible] [illegible] orifices. This type of [illegible] of constant area. An [illegible] able area over the cushio[illegible] The damping force als[illegible] [illegible] Consequent[illegible] [illegible] force. Als[illegible], it [illegible] piston displacement or [illegible] ining the orifice [illegible] tor. This equation [illegible]

Fro[illegible] the quantity of an incompressible [illegible] which is moving with a velocity [illegible] fluid under a head $h(s,x)$ flowing [illegible] and a discharge coefficient C_d. For [illegible] mping, Bernoulli's equation [17] can be used [illegible]

$$[illegible] \, (r, x)]^2 \tag{12}$$

[illegible]

[illegible] response [illegible] area can be obtained for any [illegible] conditions [illegible] form

$$[illegible] \left(\frac{[illegible] A^3}{[illegible]} \right)^{[illegible]} \left[\frac{1 - x/X}{[illegible] - S(n,x)} \right]^{[illegible]} \qquad ([illegible]6)$$

[illegible]

[illegible] for the [illegible]

$$[\text{illegible}] \left[[\text{illegible}] \right]^{\frac{1}{2}} (1 - [\text{illegible}]) \quad ([\text{illegible}])$$

[illegible]

$$[\text{illegible}] \quad ([\text{illegible}])$$

[illegible] membe [illegible] = nW/g [illegible] displacement and time [illegible] for the [illegible] are given by

$$[\text{illegible}] = \left(\frac{MgXpA^3}{C_d^2}\right)^{\frac{1}{2}} \left[\frac{1 - x/X}{[\text{illegible}] + (n, x) - S(n, x)}\right]^{\frac{1}{2}} \quad (19)$$

and

$$[\text{illegible}] = \left(\frac{MgXpA^3}{C_d^2}\right)^{\frac{1}{2}} \left[\frac{1}{nNW + K(n,t) - S(n, t)}\right]^{\frac{1}{2}} (1 - t/T) \quad (20)$$

[illegible] at this point that for any given piston location [illegible] varies inversely as the mass with a maximum value [illegible] point of impact. For a vehicle with a load, n is [illegible] so $D(n, o)$ will always be less than $D(1,o)$. This means [illegible] weight of the vehicle must be used to determine the [illegible] orifice area required for the design of the shock absorber. This represents a contradiction to the present normal design practice.

[illegible] the shock absorber controls the cushion [illegible] optimized for any value of a when either [illegible]) is satisfied. From these equations it [illegible] must be a variable which is a regulated [illegible] ment or time after impact. The shock [illegible] used as the regulated variable of [illegible] control shown in Figures 2 and 3.

[illegible] a preload is now selected for the [illegible] (9) and (20) may be used to determine th[illegible] ponse of the regulated variable that will be required for optimum control of the shock absorber cushion force. Since the required response will vary with each design, the requirements for a number of specific cases with design interests will now be presented.

[illegible]eliminary Design Examples

[illegible]

[illegible] external force is acting in the dir[illegible] [illegible] has no spring. Accordingly

[illegible]

[illegible]

[illegible]

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g X A^3}{nW}\right)^{\frac{1}{2}}(1 - x/X)^{\frac{1}{2}} \qquad [illegible]$$

[illegible] becomes

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g X A^3}{nW}\right)^{\frac{1}{2}}(1 - t/T) \qquad [illegible]$$

[illegible]

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g X A^3}{nW}\right)^{\frac{1}{2}} \qquad [illegible]$$

[illegible]

$$[illegible] = \frac{1}{C_1}\left(\frac{\rho g X A^3}{W}\right)^{\frac{1}{2}} \qquad [illegible]$$

When equation I(19) is divided by equation I(21), the equation f[illegible] displacement response of the orifice area is obtained

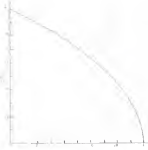
$$\frac{O(n,x)}{O(n,o)} = (1 - x/X)^{\frac{1}{2}} \qquad I(23)$$

[illegible]hen equation I(20) is divided by equation I(21), the equation for the time response of the orifice area is obtained

$$\frac{O(n,t)}{O(n,o)} = (1 - t/T) \qquad I(24)$$

[illegible] are dimensionless. They are plotted in Figures

[illegible]



... orifice area versus displacement for

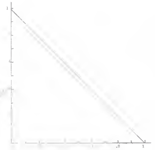


Figure 5 Dimensionless port ... area versus velocity for design example Case 1

[illegible] No external force is acting in the direction of motion. [illegible] spring is linear and has a spring constant k with no pre-[illegible]

$$[illegible] \quad 0$$

$$[illegible] \quad kx \qquad \text{where } k = nW/X$$

[illegible]

$$[illegible] \frac{1}{C_d}\left(\frac{\rho g X A^3}{W}\right)\left(\frac{1 - x/X}{n - x/X}\right)^{\frac{1}{2}} \qquad \text{II(19)}$$

[illegible]

$$[illegible] \frac{1}{C_d}\left(\frac{\rho g X A^3}{W}\right)\left[\frac{1}{n - (t/T)(2 - t/T)}\right]^{\frac{1}{2}}(1 - t/T) \qquad \text{II(20)}$$

[illegible]

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g X A^3}{nW}\right)^{\frac{1}{2}} \qquad \text{II(21)}$$

[illegible] was selected to make the orifice area a constant [illegible]pth when n = 1. Therefore

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g X A^3}{W}\right)^{\frac{1}{2}} \qquad \text{II(22)}$$

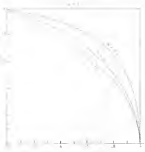
[illegible] II(19) is divided by equation II(21), the equation for [illegible] response of the orifice area is obtained.

$$\frac{0(n,x)}{0(n,o)} = \left[\frac{n(1 - x/X)}{n - x/X}\right]^{\frac{1}{2}} \qquad \text{II(23)}$$

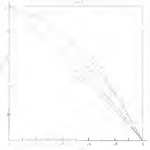
[illegible] equation II(20) is divided by equation II(21), the equation for [illegible] response of the orifice area is obtained.

$$\frac{0(n,t)}{0(n,o)} = \left[\frac{n}{n - (t/T)(2 - t/T)}\right]^{\frac{1}{2}}(1 - t/T) \qquad \text{II(24)}$$

[illegible] last two equations are dimensionless. They are plotted on [illegible] 7.



... crack ... displacement for ...



[illegible] versus velocity for design [illegible]

[illegible]

[illegible]

[illegible] where $\lambda = W/2X$

[illegible]

$$[illegible] \left(\frac{\rho g N X A^3}{W}\right)^{\frac{1}{2}} \left[\frac{(1 - x/X)}{nN - 0.1 - 0.5(x/X)}\right]^{\frac{1}{2}} \qquad \text{III(19)}$$

[illegible]

$$[illegible] \left(\frac{\rho g N X A^3}{W}\right)^{\frac{1}{2}} \left[\frac{(1 - t/T)^2}{nN - 0.1 - 0.5(t/T)(2 - t/T)}\right]^{\frac{1}{2}} \qquad \text{III(20)}$$

[illegible]

$$[illegible] \left[\frac{[illegible] NXA^3}{[illegible] N - 0.1)}\right]^{\frac{1}{2}} \qquad \text{III(21)}$$

[illegible]

$$[illegible] \left[\frac{\rho g N X A^3}{W(N - 0.1)}\right]^{\frac{1}{2}} \qquad \text{III(22)}$$

When eq[illegible] is divided by equation III(21), the equation for the displ[illegible] se of the orifice area is obtained

$$[illegible] = \left[\frac{(nN - 0.1)(1 - x/X)}{nN - 0.1 - 0.5x/X}\right]^{\frac{1}{2}} \qquad \text{III(23)}$$

When eq[illegible]I(20) is divided by equation III(21), the equation for [illegible] f the orifice area is obtained

$$[illegible] \left[\frac{nN - 0.1}{nN - 0.1 - 0.5(t/T)(2 - t/T)}\right]^{\frac{1}{2}} (1 - t/T) \qquad \text{III(25)}$$

[illegible] be equal to 1.4 for an impact velocity of 15 fps and a [illegible] of 2-5 feet. The last two equations are dimensionless [illegible] for N = 1.4 in Figures 8 and 9.

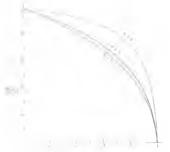
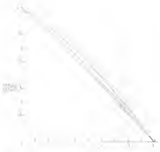


Figure 8 Dimensionless plot of orifice area versus displacement design example Case III.



[illegible]

[illegible] [illegible] [illegible] has no spring

$$f(n,x) = nW$$

$$S(n, x) = 0$$

[illegible] (19) becomes

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g N X A^3}{nW(N+1)}\right)^{\frac{1}{3}} [illegible]$$ [illegible]

[illegible] (19) becomes

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g N X A^3}{nW(N+1)}\right)^{\frac{1}{3}} [illegible]$$ IV(20)

[illegible]

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g N X A^3}{nW(N+1)}\right)^{\frac{1}{3}}$$ IV(21)

[illegible]

$$[illegible] = \frac{1}{C_d}\left(\frac{\rho g N X A^3}{W(N+1)}\right)^{\frac{1}{3}}$$ IV(22)

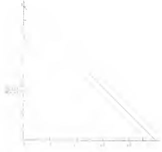
[illegible] equation IV(19) is divided by equation IV(21), the equation for [illegible] response of the [illegible] is obtained

$$[illegible] = (1 - x/\lambda)^2$$ IV(23)

[illegible] divided [illegible] equation IV(22), the equation for [illegible] is [illegible] obtained

$$[illegible]$$ IV(24)

[illegible] identical to those of Case I. Since the [illegible] which they were derived are different, they are plotted in Figures 10 and 11.



[illegible] The [illegible] to the [illegible] and [illegible] [illegible] of spring [illegible] at [illegible]. The spring [illegible] to produce

[illegible]

[illegible] [illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]. Thus

$$[illegible]$$

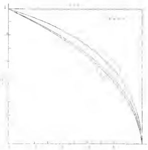
[illegible], the equation for the [illegible] is obtained.

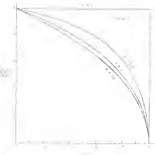
$$[illegible] \; (1 - x/l)^{\frac{1}{2}} \qquad V(2[illegible])$$

[illegible] equation V(21), the equation for the time [illegible] is obtained.

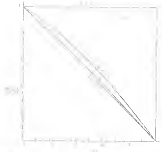
$$[illegible] \frac{1}{(1 - t/T)} \Big]^{\frac{1}{2}} (1 - t/T) \quad [illegible]$$

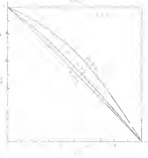
[illegible] They are plotted [illegible] [illegible] 13, 14 and 15.





[illegible] versus displacement
[illegible]





[illegible] constant k. [illegible] spring [illegible]

[illegible]

[illegible] VI(19)

[illegible] VI(20)

and

[illegible] VI(21)

[illegible] constant [illegible] the orifice [illegible] length [illegible]. Therefore,

$$[illegible] \left(\frac{\rho g X A^2}{2} \right) \qquad \text{VI(22)}$$

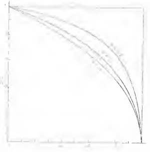
[illegible] (19) [illegible] divided [illegible] equation VI(21), the equation for [illegible] obtained.

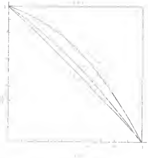
[illegible] VI(23)

[illegible] the equation for [illegible]

[illegible] VI(2[illegible])

[illegible] but the [illegible] to the spring [illegible] is equal to $X + W(n-1)/k$ [illegible] 17.





frontal [illegible] assumed the maximum hydrau[illegible]

[illegible]

[illegible] seen [illegible]

[illegible] piston [illegible]

[illegible]

Commercial hydraulic fluids with no entrapped air may be considered incompressible for pressures less than 10,000 psi [4], the fluid will

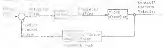
impact the required time rate of change of the regulated variable ... orifice will then be

$$\frac{O(1,o) - O(3,o)}{0.067} = 50\,(0.0+0 - 0.021)$$

$$d[O(1,o)]/dt = 0 \ldots \text{ ft/sec}$$

... the leakage ... orifice length ... curves of Figures ... regulated variable ... application.

... regulator will now be designed. The optimum ... (11) will be used as the controlled output. A ... hydraulic actuator with a reference ... variable orifice of a



[illegible] th Closed Loop Regulator

[illegible] absorber and regulator is shown [illegible] both pistons have the same [illegible] and velocity

[illegible] the regulator piston pressure [illegible] velocity satisfies equation (11). An [illegible] orifice [illegible] then be used for the design of the [illegible] piston. This variable orifice will be designed [illegible] piston and an optimum hydraulic pressure of [illegible] the [illegible] piston is

[illegible]

[illegible] of the regulator cylinder. The [illegible] plus the [illegible] of the [illegible] of the orifice [illegible] regulator cylinder is determined from equation [illegible]

[illegible]

[illegible] the [illegible] position [illegible] is now used [illegible] pressure was increased from [illegible] impact

[illegible] to regulate the orifice [illegible] The valve has a piston [illegible] 0.3125 in. and a weight of [illegible]

[illegible] 5 [illegible] with 50 turns [illegible] of 10 lb./in. and [illegible]

[illegible] valve [illegible] 108 slug

[illegible] and spring force [illegible] will be

$$\left[\ldots \right] = -\frac{12}{17}$$

[illegible]

[illegible] response at impact of the regulator [illegible] using a spring loaded spool valve

[illegible] area [illegible]

[illegible] critical [illegible]

[illegible] will [illegible]

[illegible] and measurement will be

[illegible] the cushion force

[illegible] optimum. The shock

[illegible] change in decele-

[illegible] time. The final design

of the [illegible] therefore consider instability, frequency response,

[illegible] inherent damping.

SUMMARY AND DI[illegible]ON

[illegible] that after im[illegible] [illegible] a constant [illegible] [illegible] [illegible] to more than one value of mass [illegible] [illegible] of a hydraulic type shock ab[illegible] [illegible] regulated orifice area are derived [illegible] [illegible] after impact. At impact, the [illegible] [illegible]ly as the vehicle mass.

[illegible] control is used for self-regulatio[illegible] [illegible] The c[illegible] output of this regulator is the [illegible] railroad [illegible] car shock absorber with self-re[illegible] [illegible] vehicle [illegible] investi[illegible]ted. After impact, a [illegible] [illegible] required to adjust the ori-[illegible] [illegible] of the [illegible] orifice should be [illegible] [illegible] of this type.

[illegible] as the controlled [illegible] [illegible] and the experie[illegible] of the [illegible] the necessary frequency re[illegible] [illegible] variation of the constant deceleration re-[illegible] [illegible] optimum shock absorber response.

[illegible]

1. [illegible]

2. [illegible] Two-Line [illegible]

3. [illegible]

4. [illegible]

5. [illegible] Shock [illegible] 168 – 1[illegible]

6. [illegible] Designing Shock [illegible] 4, pp. 217 [illegible]

7. [illegible] "[illegible]namic [illegible] Novel High [illegible] Freight [illegible] of Engineering [illegible] 81 58[illegible]

8. [illegible] Conway "The Aero[illegible] Undercarriage" Proceedings, [illegible] Mechanical Engineers, Vol. 156 19[illegible], pp. 272-281.

9. [illegible] "Comparison of Linear and Nonlinear Dampers for [illegible]" Journal of Aircraft, Vol. 15, October 1978, pp. 696 [illegible]

10. [illegible] Impact Absorber with Two-Stage, Variable Area Orifice [illegible] Damper" Journal of Sound and Vibration, Vol. 50, Jan. 2[illegible] 19[illegible] 702-[illegible]

11. [illegible] Analysis of Performance of Pneumatic Impact Absorber [illegible] for [illegible] Sept. 26, 1977

12. [illegible] the Polytropic Air Compression [illegible] Shock Struts" Journal of Aircraft [illegible] 1976, pp. [illegible]-530

13. [illegible] Machine Design, September 18, 1980, Vol. [illegible]

[illegible]

[illegible]

[illegible] Williams, [illegible] [illegible] York, 1967 [illegible]

[illegible] Holt and [illegible] [illegible]

BIOGRAPHICAL SKETCH

[illegible] [illegible] [illegible]aman was born in Montev[illegible] Minnesota, [illegible] [illegible] [illegible] the Montevideo Central High School, he [illegible] [illegible] [illegible] [illegible]gineering program of the University of Minnesota and [illegible] [illegible] in 1938. He later received a MSAE from the Univer[illegible] [illegible] [illegible] 1951 and a MSCE from Colorado State University in 19[illegible].

[illegible], he began doctoral studies in the Engineering Scien[illegible] Department [illegible] the University of Florida. His dissertation research was in the [illegible] of impact absorber optimization. He expects to g[illegible]uate in December, 1983, with a Ph.D. in aerospace engineering.

Mr. [illegible]eeman is a Registered Professional Engineer with thirty-five years of engineering teaching and industrial experience.

He has been married for thirty-eight years to the former Hele[illegible] Krause of St. Albans, New York and has four children.

I certify that I have [illegible] conforms to acceptable sta[illegible] adequate, in scope and qual[illegible] Doctor of Philosophy.

Dr. Knox Millsaps, Chairman
Professor of Engineering Science

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Dr. Richard L. Fearn
Professor of Engineering Sciences

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Dr. Mark H. Clarkson
Professor of Engineering Sciences

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Professor of Engineering Sciences

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy

Charles A. Burnap

Dr. Charles A. Burnap
Assistant Professor of Mathematics

This dissertation was submitted to the Graduate Faculty of the College of Engineering and to the Graduate Council and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy

J W Justman

[illegible] Engineering

Dec. 1980

Dean, Graduate Studies and Research